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The Minimum Gap-opening Planet Mass in an Irradiated Circumstellar Accretion Disk

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ABSTRACT
We consider the minimum mass planet, as a function of radius, that is capable of opening a gap in an α-accretion disc. We estimate that a half Jupiter mass planet can open a gap in a disc with accretion rate $\dot{M} \lesssim 10^{-8} M_\odot$/yr for viscosity parameter $\alpha = 0.01$, and Solar mass and luminosity. The minimum mass is approximately proportional to $\dot{M}^{0.48}\alpha^{0.8}M_*^{0.42}L_*^{-0.08}$. This estimate can be used to rule out the presence of massive planets in gapless accretion discs. We identify two radii at which an inwardly migrating planet may become able to open a gap and so slow its migration; the radius at which the heating from viscous dissipation is similar to that from stellar radiation in a flared disk, and the radius at which the disc becomes optically thin in a self-shadowed disc. In the inner portions of the disc, we find that the minimum planet mass required to open a gap is only weakly dependent on radius. If a migrating planet is unable to open a gap by the time it reaches either of the transition radii, then it is likely to be lost onto the star. If a gap opening planet cuts off disc accretion allowing the formation of a central hole or clearing in the disc then we would estimate that the clearing radius would approximately be proportional to the stellar mass.

Key words:

1 INTRODUCTION

Recent observations have identified young (1–3 Myr old) stars that have inner clearings in the dust distribution as inferred from their IRS spectra (CoKuTau/4, D'Alessio et al. 2005; GM Aur, TW Hya and DM Tau , Calvet et al. 2002, 2005, and brown dwarf candidates L316 and L30003 in IC 348, Muzerolle et al. 2006). These disks have been dubbed "transitional disks" as they represent the stage in which circumstellar disks are probably disappearing and in which massive planets are thought to be forming. In some cases there is still accretion on to the central star; DM Tau and GM Aur at rates of 2×10^{-9} and $10^{-8} M_\odot$/yr, respectively. In other cases there is no evidence for accretion, i.e., CoKuTau/4, even though its outer disk resembles that of other T-Tauri stars. Multi-band IRAC/MIPS photometric measurement from imaging surveys have been used to identify dozens of other transition disk candidates (e.g. Sicilia-Aguilar et al. 2006). Candidates are then observed with IRS allowing accurate measurements of edge wall heights, and temperatures and so radii (e.g. Kim 2007).

Two dominant approaches exist toward predicting and accounting for holes or clearings in young circumstellar disks. The first is planet formation followed by an opening of a gap and subsequent clearing of an inner disk (e.g. Quillen et al. 2004; Varnière et al. 2006; Crida and Morbidelli 2007). Alternatively, photo-evaporation (e.g. Clarke et al. 2001; Alexander et al. 2006) may clear the disc from the inside out. A third possibility is that smaller dust particles are preferentially destroyed at a particular radius due to turbulence (Ciesla 2007), within the context of a dead zone model. Photo-evaporation models can account for large disk clearings near luminous stars (Kim 2007) and non-accreting systems such as CoKuTau/4. However they are not sufficiently sophisticated that they can account for clearings in disks hosted by low luminosity brown dwarfs (UV flux is too low, Muzerolle et al. 2006)), disks with wide radial gaps and inner disks (such as GM Aur) or disks that are truncated in the dust distribution but continue to accrete (such as DM Tau). Recently, Chiang and Murray-Clay (2007) have suggested that stellar X-rays would be able to ionise the inner wall of a gap enough to trigger accretion via the magneto-rotational instability. The disc would then clear from the inside out, at an accelerating rate. Models involving a planet are likely to be more versatile but because of the complexity of planet/disk interactions this scenario has been little explored. In this paper we focus on the possibility that massive planets open gaps in disks and explore what mass planets are capable of opening gaps in different accretion disks.

Because a planet drives density waves into a disk (Lin and Papaloizou 1979; Goldreich and Tremaine 1980) if the planet is sufficiently massive it can overcome the effect of viscosity and open a gap in the disk (Lin and Papaloizou 1993; Ward and Hahn 2000). Crida et al. (2006) have generalized the gap

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opening criterion to simultaneously take into account the mass of the planet, the scale height of the disk and the disk viscosity:

$$\frac{3}{4} \cdot \frac{H}{R_H} + \frac{50}{qR_{ey}} \leqslant 1 \quad (1)$$

where H is disk scale height, R_H is the Hill radius, R_{ey} is the effective turbulent Reynolds number and q is the mass ratio of planet mass (M_p) to stellar mass (M_*):

$$R_H \equiv r_p \left(\frac{q}{3}\right)^{1/3},\ R_{ey} \equiv \frac{r_p^2 \Omega_p}{\nu},\ q \equiv \frac{M_p}{M_*}$$

Here r_p is the planet's semi-major axis, ν is the disk viscosity and Ω_p is the angular rotation rate of an object in a circular orbit at r_p. This criterion combines two earlier conditions; the viscous condition by Bryden et al. (1999), and the tidal condition by Lin and Papaloizou (1993).

Gap opening has been studied extensively numerically with simulations (e.g. Bryden et al. 1999; Crida et al. 2006; de Val-Borro et al. 2006). However previous work has not predicted what mass planets are capable of opening gaps in circumstellar disks with different accretion rates or structure. The gap opening criterion depends on the temperature profiles of the disk through the dependence of the viscosity, ν, on the sound speed, c_s, and vertical scale height, H. Here we adopt an α form for the viscosity (Shakura and Syunyaev 1973),

$$\nu = \alpha c_s H. \quad (2)$$

Since accretion disc viscosity is poorly understood, ν and hence R_{ey} are not precisely defined in this work; this problem is common to all work on accretion discs.

The disk mid-plane temperature depends on the source of heating. We consider viscous heating and heating from absorption of radiation from the central star. Passive or non-accreting circumstellar disk models that include the effect of radiation from the central star absorbed onto the disk have been studied by Chiang and Goldreich (1997); Bell (1999); Dullemond and Dominik (2004). Because of flaring, the irradiation of the disk star dominates the disk heating at large radii when both viscous heating and that from irradiation are considered (Calvet et al. 1991; D'Alessio et al. 2001; Garaud and Lin 2007). Passive self-shadowed disks can be considerably colder and thinner at large radii (Dullemond and Dominik 2004).

In Section 2 we describe simple disk models and our procedure for estimating disk temperatures, scale heights. From these, we can derive the minimum gap-opening planet mass as a function of radius when heating is due to viscous dissipation and when heating is due to stellar irradiation. Both cases are considered separately so we can be sufficiently flexible to discuss the case of a self-shadowed disc. A disc that becomes self-shadowed will have significantly reduced heating due to stellar irradiation but heating from viscous dissipation would still be present. Section 3 presents our results, and a discussion follows in Section 4.

2 GAP-OPENING IN ACCRETION DISKS

In this section we apply the gap opening criterion (Crida et al. 2006) to the α-disk model of Shakura and Syunyaev (1973) including the effect of stellar irradiation.

The accretion rate $\dot{M}$ of a steady thin disk can be calculated using mass and angular momentum conservation

$$\dot{M} = 3\pi\Sigma\nu. \quad (3)$$

where Σ is the disc surface mass density. As the disk viscosity depends on the disk temperature we must consider sources of heat to compute it. We use the α prescription to compute the viscosity (equation 2) but compute the sound speed and disk scale height using the disk midplane temperature and the relation for hydrostatic equilibrium,

$$H = c_s/\Omega, \quad (4)$$

and the sound speed,

$$c_s = \sqrt{\frac{k_B T}{\mu m_H}} \quad (5)$$

where k_B is the Boltzmann constant, μ is the mean molecular weight and m_H is the mass of the hydrogen atom. In our calculation, we take the mean molecular weight $\mu = 2.4$ appropriate for interstellar gas.

We first compute the structure of the disk taking into account heat generated from viscous dissipation. Then we consider the case of heat generated from the radiation absorbed from starlight.

2.1 Heat generated from viscous dissipation

Dissipation due to viscosity gives the energy relation,

$$\frac{9}{8}\Sigma\Omega^2\nu = \epsilon\sigma T_{\nu,s}^4, \quad (6)$$

where ϵ is the emissivity, σ is the Stephan-Boltzmann constant. and $T_{\nu,s}$ is the temperature at the disk surface. A vertical average relates the surface temperature to the midplane temperature, $T_{\nu,c}$ by $T_{\nu,c}^4 = \frac{3}{8}\tau T_{\nu,s}^4$ where τ is the optical depth from the surface to the midplane, $\tau = \kappa\Sigma/2$, and κ is the opacity (e.g., section 2 of Armitage et al. 2001). We note that the relation between $T_{\nu,s}$ and $T_{\nu,c}$ in Equation 6 is only valid for optically thick disks. In an optically thin region, we set $T_{\nu,c}$ to $T_{\nu,s}$.

We adopt a convenient analytic form of the emissivity and opacity laws that is based on the assumption that dust grains govern the opacity and emissivity, $\epsilon = (T/T_\odot)^b$ and $\kappa = \kappa_V (T/T_\odot)^b$ where κ_V=1 cm^{2}g^{-1}, $T_\odot$ is the solar effective surface temperature and $b = 1$ (Chiang and Goldreich 1997; Garaud and Lin 2007). This form is expected as the dust temperature over much of the disk corresponds to a peak wavelength (for a black body spectrum) that is larger than the diameter of the dust particles and the diameter is of order the peak wavelength of a Solar temperature black body (e.g. Backman et al. 1992; Chiang and Goldreich 1997). Using Equations 2), 3—6 and the opacity law, we solve for the midplane temperature finding

$$T_{\nu,c} \approx \begin{cases} \left[\frac{3}{128\pi^2}\frac{\mu m_H}{k_B}\frac{\kappa_V}{\sigma\alpha}\Omega^3\dot{M}^2\right]^{\frac{1}{5}} & \text{for } \tau \gg 1 \\ \left[\frac{3}{8\pi}\frac{\dot{M}\Omega^2 T_\odot}{\sigma}\right]^{\frac{1}{5}} & \text{for } \tau \ll 1 \end{cases} \quad (7)$$

The opacity drops to the low opacity regime at an approximate dividing line of $\tau = 8/3$ at a radius

$$\begin{aligned} r_\tau &\sim \left[\frac{\kappa_V}{16\pi}\frac{\mu m_H}{k_B T_\odot}\frac{\dot{M}}{\alpha}\right]^{\frac{2}{3}} (GM_*)^{\frac{1}{3}} \\ &\sim 1.2\,\mathrm{AU}\left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{\frac{2}{3}}\left(\frac{\alpha}{0.01}\right)^{-\frac{2}{3}}\left(\frac{M_*}{M_\odot}\right)^{\frac{1}{3}} \end{aligned} \quad (8)$$

For the α disk model, using the above equations, we solve for all disk variables (temperature, density, viscosity, scale height) as functions of parameters α, $\dot{M}$, and M_* and as a function of radius, r.

2.2 Heating due to irradiation from the star

The disk can be heated not only by disk viscosity but also by the stellar radiation. In this section, we consider the latter case only. If we regard the central star as a point source, then the surface temperature depends on the slope of the disk,

$$\frac{L_*}{4\pi r^2}(1-\beta)\frac{H}{r}\left(\frac{\mathrm{d}\ln H}{\mathrm{d}\ln r}-1\right)=\epsilon\sigma T_{i,s}^4, \qquad (9)$$

(Frank et al. 2002), where L_* is the stellar luminosity, β is the albedo and $T_{i,s}$ is disk surface temperature by irradiation.

Here, we assume that the disk temperature does not strongly depend on height, though a more detailed model would more carefully compute the disk structure (e.g. D'Alessio et al. 1998; Garaud and Lin 2007). Indeed, if the disc is very optically thick, then the midplane temperature will always be dominated by viscous dissipation (Dubus et al. 1999). Ignoring the vertical temperature gradient, we estimate the disk midplane temperature $T_{i,c} \simeq T_{i,s}$ using Equations 3—6 and 9:

$$T_{i,c} \sim \left(\frac{AT_\odot}{\sigma}\right)^{\frac{2}{9}}\left(\frac{k_B}{\mu m_H}\right)^{\frac{1}{9}}(GM_*)^{-\frac{1}{9}}\, r^{-\frac{1}{3}} \qquad (10)$$

where the coefficient

$$A=\frac{L_*}{4\pi}(1-\beta)\left(\frac{\mathrm{d}\ln H}{\mathrm{d}\ln r}-1\right)$$

We restrict our solution for the disk scale height to a self-consistent power law form with $d\ln H/d\ln r$ equal to a constant. We find that the disk scale height

$$H \sim \left(\frac{L_*(1-\beta)T_\odot}{12\pi\sigma}\right)^{\frac{1}{9}}\left(\frac{k_B}{\mu m_H}\right)^{\frac{5}{9}}(GM_*)^{-\frac{5}{9}} r^{\frac{4}{3}}, \qquad (11)$$

and that $d\ln H/d\ln r = 4/3$. If we consider an albedo in the range $0<\beta<0.5$ (Wood et al. 2002), then Equations 10 and 11 imply that the disk structure is not strongly sensitive to the albedo. In our subsequent estimates, we have adopted an albedo of $\beta=0$.

Interior to a particular radius, the heat from viscous heating dominates that from stellar irradiation. Setting the heating rate due to accretion equal to that due to irradiation (setting the left hand side of Equation 6 to the left hand side of Equation 10 and solving for radius) we estimate a transition radius

$$r_{tr} \sim \left(\frac{9}{2}\right)^{\frac{3}{4}}\left(\frac{12\pi\sigma}{T_\odot}\right)^{\frac{1}{12}}(L_*(1-\beta))^{-\frac{5}{6}} \times$$
$$\left(\frac{\mu m_H}{k_B}\right)^{\frac{5}{12}}\dot{M}^{\frac{3}{4}}(GM_*)^{\frac{7}{6}} \qquad (12)$$
$$\approx 0.3\mathrm{AU}\left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{\frac{3}{4}}\left(\frac{M_*}{M_\odot}\right)^{\frac{7}{6}}\left(\frac{L_*}{L_\odot}\right)^{-\frac{5}{6}}$$

For $r<r_{tr}$ we expect that viscous heating would dominate.

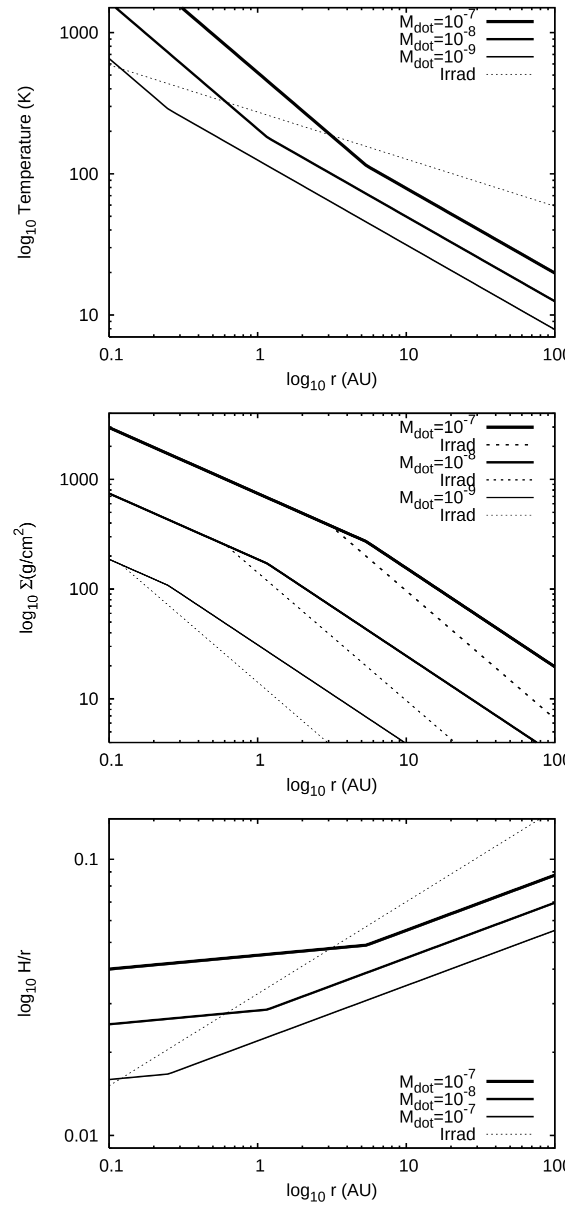

Figure 1. a) Disk radius vs. midplane temperature for disk mass accretion rates $\dot{M}=10^{-9},10^{-8}$ and 10^{-7} $M_\odot$/yr for $\alpha=0.01$ and stellar mass $M_*=1M_\odot$. The solid lines are for the case of viscous heating only and the dotted lines are for the case of irradiated heating only and $L_*=L_\odot$. A comparison between the dotted and solid lines shows that heating due to stellar radiation dominates at large radius. A change in slope in the solid lines occurs at the radius where the disc becomes optically thin (r_τ). b) Similar to a) except the surface density profile is shown. c) Similar to a) except the aspect ratio H/r is shown.

3 RESULTS

In Figure 1 we show disk variables as a function of radius for different accretion rates $\dot{M}=10^{-7},10^{-8}$, and $10^{-9}M_\odot$/yr but fixed $\alpha=0.01$ and stellar mass, $M_*=1M_\odot$. Figure 1a shows the disk mid-plane temperature, Figure 1b shows the surface density profile and Figure 1c the vertical scale height for these disks, and as a function of radius in each case. In our Figures, we show with solid lines the variables calculated for heating primarily from viscous dissipation while variables primarily due to the stellar radiation are shown with dotted lines. A comparison between the dotted and solid lines shows that heating due to stellar radiation dominates at large radius whereas heating due to viscous dissipation dominates at small radius. In Figure 1a-c the change in slope in the solid lines occurs where the disc becomes optically thin at a radius approximately given by r_τ (Equation 8).

We solve for the minimum gap opening planet mass ratio using Equation 1. With a variable substitution of $y=(q_0/q)^{\frac{1}{3}}$ equa-

tion 1 can be written as a cubic equation

$$By + y^3 = 1$$

where we have defined

$$q_0 \equiv \frac{50}{R_{ey}},\ B = \frac{3}{4} \cdot \left(\frac{3h}{50\alpha r}\right)^{\frac{1}{3}}$$

The cubic equation has only one real root [1]

$$y = \frac{B}{3u} - u$$

with

$$u^3 = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{B^3}{27}}.$$

For the entire parameter space covered here we find that $B < 1$ and the minimum gap opening planet mass $q_{min} \approx q_0$. Although the prediction of Crida et al. (Equation 1 in this paper) is more general, we find that for most disc models the viscous condition of previous work suffices to estimate the minimum gap opening planet mass. We would not expect this to be the case in discs containing 'dead zones' (Gammie 1996), where the viscosity is low, but the scale height remains large.

Figure 2 shows the minimum gap-opening planet mass ratio for these disk models as a function of radius. In the inner regions, where viscous heating dominates, the minimum q required to open a gap is not strongly dependent on radius. However, at larger radii, the minimum gap opening q begins to rise more steeply with increasing radius. This happens for both self-shadowed and irradiated discs. If the disc is self-shadowed, then we would expect the minimum planet mass for gap opening always to lie along the solid (viscous heating) line. As noted in Section 2, Dubus et al. (1999) showed that the midplane temperature of a very optically thick irradiated disc would be dominated by viscous dissipation. In this case, the minimum planet mass required to open a gap would lie somewhere between the solid (viscous) and dotted (irradiated) lines. However, unless the disc is self-shadowed, the curve must rejoin the dotted line around r_τ, since then the radiation will be able to penetrate to the midplane.

Figure 2 may be compared to figure 7 of Menou and Goodman (2004). They were particularly interested in Type I migration rates on T Tauri discs, and as part of this, estimated the minimum gap opening planet in their models. Our model is showing similar behaviour, with the minimum mass required for gap opening relatively flat out to $\sim 10\,$au, and rising thereafter. This is despite the numerous differences in the disc and dust models, and the gap opening criteria used. Jang-Condell and Sasselov (2005) also noted that the minimum planet mass required to open a gap would be lower nearer to the star.

Because the minimum gap opening planet mass ratio is less sensitive to radius in the inner regions, the transition radius (r_{tr}, Equation 12), between viscous heating and heating by stellar radiation sets a favourable spot for a planet migrating inward via type I migration to slow its migration rate. Type II migration, following gap opening, is expected to be slower than type I migration for Earth mass objects, lacking a gap (Ward and Hahn 2000). A planet migrating in the outer disk could become able to open a gap as it moved inward. The planets that survive would perhaps be the ones that were just massive enough to open gaps in the disc at this transition radius; planets unable to open gaps would continue to migrate

[1] See `http://mathworld.wolfram.com/CubicFormula.html`

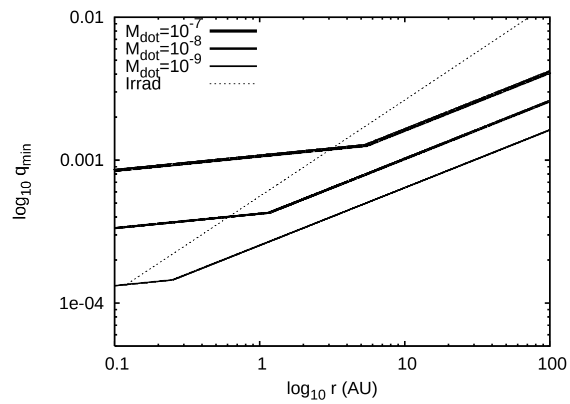

Figure 2. The minimum gap-opening planet mass ratio as a function of radius (computed using Equation 1) for the disks shown in Figure 1 with accretion rates $\dot{M} = 10^{-7}, 10^{-8}$, and $10^{-9} M_\odot$/yr. The transition radius is evident where the dotted line intersects the solid lines. This radius is where midplane temperature from viscous dissipation is similar to that from stellar radiation. Inside this radius the minimum gap-opening planet mass is not strongly sensitive to radius. Outside this radius a larger planet mass is required to open a gap, as long as the disk is sufficiently flared to be heated by starlight. Lower mass planets can open a gap at the larger transition radius (slope change of the solid lines) in self-shadowed disks.

rapidly, and be lost onto the star. If a flared disk does not contain a gap, then the minimum gap opening planet mass ratio computed at this transition radius, r_{tr} (Equation 12) provides an estimate for its minimum mass.

As the minimum gap opening planet mass is approximately equal to q_0, we can compute how it scales with radius, stellar mass, luminosity, accretion rate and α parameter. For heating from viscous dissipation and an optically thick disc, the minimum gap opening planet mass ratio is

$$\begin{aligned} q_{min,\nu,1} &\sim 50\left(\frac{\alpha k_B}{\mu m_H}\right)^{\frac{4}{5}}\left(\frac{3\kappa_V \dot{M}^2}{128\pi^2\sigma}\right)^{\frac{1}{5}}(GM_*)^{-\frac{7}{10}}\, r^{\frac{1}{10}} \\ &\sim 4\times10^{-4}\left(\frac{\alpha}{0.01}\right)^{\frac{4}{5}}\left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{\frac{2}{5}}\times \\ &\left(\frac{M_*}{M_\odot}\right)^{-\frac{7}{10}}\left(\frac{r}{\mathrm{AU}}\right)^{\frac{1}{10}} \end{aligned} \quad (13)$$

The minimum gap opening planet mass ratio is only weakly dependent on radius (with exponent $1/10$) and most strongly dependent on α and M_*. For heating from viscous dissipation and an optically thin disc, the minimum gap opening planet mass ratio

$$\begin{aligned} q_{min,\nu,0} &\sim 50\alpha\left(\frac{k_B}{\mu m_H}\right)\left(\frac{3\dot{M}T_\odot}{8\pi\sigma}\right)^{\frac{1}{5}}(GM_*)^{-\frac{4}{5}}\, r^{\frac{2}{5}} \\ &\sim 4\times10^{-4}\left(\frac{\alpha}{0.01}\right)\left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{\frac{1}{5}}\times \\ &\left(\frac{M_*}{M_\odot}\right)^{-\frac{4}{5}}\left(\frac{r}{\mathrm{AU}}\right)^{\frac{2}{5}} \end{aligned} \quad (14)$$

The minimum gap opening planet mass ratio is more strongly dependent on radius when the disk is optically thin than optically thick.

When heating is due to irradiation by the star, we find

$$q_{min,i} \sim 50\alpha \left(\frac{k_B}{\mu m_H}\right)^{\frac{10}{9}} \left(\frac{L_*(1-\beta)T_\odot}{12\pi\sigma}\right)^{\frac{2}{9}} \times (GM_*)^{-\frac{10}{9}} r^{\frac{2}{3}} \sim 6\times 10^{-4} \left(\frac{\alpha}{0.01}\right) \left(\frac{L_*}{L_\odot}\right)^{\frac{2}{9}} \left(\frac{M_*}{M_\odot}\right)^{-\frac{10}{9}} \left(\frac{r}{\mathrm{AU}}\right)^{\frac{2}{3}} \quad (15)$$

The radial exponent here is 2/3, larger than for either the high and low opacity viscous heating cases. For the irradiated disk q_{min} depends upon the α parameter because the gap opening criterion depends on the disk viscosity, however q_{min} does not depend on the accretion rate $\dot{M}$ because the disk temperature is independent of $\dot{M}$.

Were the disk to become self-shadowed then the solid lines on Figure 2 and subsequent figures are relevant. The minimum gap opening planet mass does not significantly change, but the radius at which a low mass object can open a gap is further out and set instead by the location of r_τ (Equation 8). If the disk is self-shadowed then the radius at which the disk becomes optically thin, r_τ, might set a favorable spot for an inward migrating core to open a gap.

We compute the minimum gap opening planet mass ratio for an optically thick disc at the transition radius r_{tr} finding

$$q_{min}(r_{tr}) \sim 4\times 10^{-4} \left(\frac{\alpha}{0.01}\right)^{0.8} \left(\frac{L_*}{L_\odot}\right)^{-0.08} \times \left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{0.48} \left(\frac{M_*}{M_\odot}\right)^{-0.58} \quad (16)$$

This can be used to place a constraint on possible planets residing in a gapless disk,

$$M_p \lesssim 0.4\, M_J \left(\frac{\alpha}{0.01}\right)^{0.8} \left(\frac{L_*}{L_\odot}\right)^{-0.08} \times \left(\frac{\dot{M}}{10^{-8}M_\odot/\mathrm{yr}}\right)^{0.48} \left(\frac{M_*}{M_\odot}\right)^{0.42} \quad (17)$$

where M_J is a Jupiter mass. The above minimum gap opening planet mass ratio and mass are appropriate for both flared and self-shadowed disks because the minimum gap opening planet mass is insensitive to radius within the transition radius.

In Figure 3 we show the minimum gap-opening planet mass ratio for three disks with the same accretion rate, $\dot{M} = 10^{-8}M_\odot$/yr, and stellar mass, $M_* = 1M_\odot$. but for three different α parameters, $\alpha = 0.1, 0.01$ and 0.001. Figure 3 can be directly compared to Figure 2 as the central solid and dotted lines are the same. We see that lower mass planets can open a gap in lower α disks. We note that the minimum gap opening planet mass is more strongly sensitive to this poorly constrained parameter, α than the other parameters, such as stellar mass, luminosity and accretion rate.

In Figure 4 we show the minimum gap-opening planet mass ratio for three disks with the same accretion rate, $\dot{M} = 10^{-8}M_\odot$/yr, and $\alpha = 0.01$ but with different stellar masses, $M_* = 0.5, 1.0$ and $2.0M_\odot$. Figure 4 can be directly compared to Figures 2 and 3. The luminosity of a star depends on the stellar mass, and here we have assumed that $L_* = L_\odot(M_*/M_\odot)^3$ to take this into account. Figure 4 shows that the lower mass stars (rather than higher mass ones) require higher planet mass ratios to open a gap.

Figure 5 shows that the minimum planet mass ratio for three

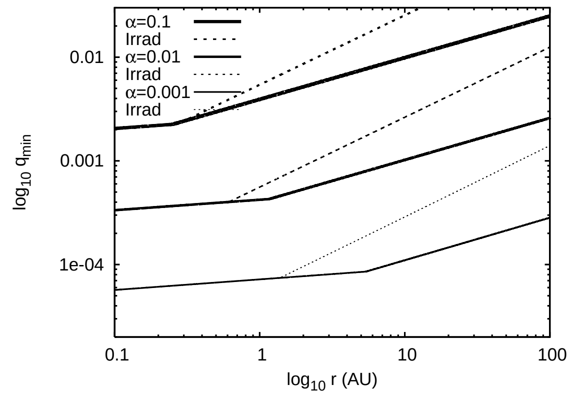

Figure 3. Similar to Figure 2 except the minimum gap-opening planet mass is shown as a function of radius for a disk with $\alpha = 0.001, 0.01, 0.1$, with accretion rate $\dot{M} = 10^{-8}M_\odot$/yr and stellar mass $M_* = 1M_\odot$. The solid lines are for the case of viscous heating only and the dotted lines are for the case of irradiated heating only.

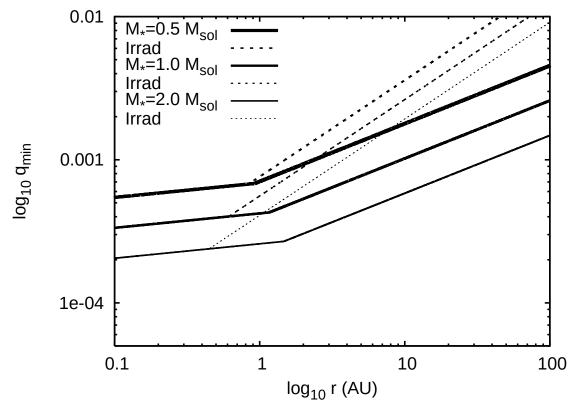

Figure 4. Similar to Figure 2 and 3 except the minimum gap-opening planet ratio for different stellar masses $M_* = 0.5, 1, 2\ M_\odot$ for a disk with $\alpha = 0.01$ and accretion rate $\dot{M} = 10^{-8}M_\odot$/yr. The solid lines are for the case of viscous heating only and the dotted lines are for the case of irradiated heating only. We have assumed that $L_* = L_\odot(M_*/M_\odot)^3$.

different mass stars with the additional requirement that the accretion rate is proportional to the stellar mass $\dot{M} \propto M_*^2$, as suggested by observational surveys (Muzerolle et al. 2005). With this assumption we find that the minimum gap opening planet mass ratio is approximately independent of stellar mass. Assuming that stellar luminosity scales with mass, $L_* = L_\odot(M/M_*)^3$ and $\dot{M} = 10^{-8}M_\odot/\mathrm{yr}\left(\frac{M_*}{M_\odot}\right)^2$, Equation 17 becomes

$$M_p \lesssim 0.4 M_J \left(\frac{\alpha}{0.01}\right)^{0.8} \left(\frac{M_*}{M_\odot}\right)^{1.14} \quad (18)$$

leaving only α as an undetermined parameter. If larger planets are formed earlier during epochs of higher accretion around more massive stars then we would predict that they might be more likely to be located at larger radii. This follows because the transition radius is larger for higher mass stars (Equation 12) as is r_τ (Equation 8).

The favorable location for an inward migrating planet to migrate inward for a flared disk assuming $L_* = L_\odot(M/M_*)^3$ and $\dot{M} = 10^{-8}M_\odot/\mathrm{yr}\left(\frac{M_*}{M_\odot}\right)^2$ would be the transition radius com-

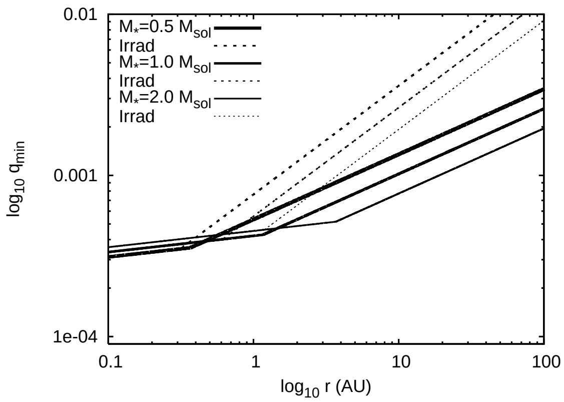

Figure 5. The minimum planet mass ratio for stellar masses $M_* = 0.5, 1, 2\ M_\odot$ for disks with $\alpha = 0.01$. We have assumed that $\dot{M} = 10^{-8}\ M_\odot/\mathrm{yr}\ (M_*/M_\odot)^2$ and $L_* = L_\odot(M_*/M_\odot)^3$. The solid lines are for the case of viscous heating only and the dotted lines are for the case of irradiated heating only.

puted for these assumptions

$$r_{tr} \sim 0.3\mathrm{AU}\left(\frac{M_*}{M_\odot}\right)^{0.17} \tag{19}$$

(modifying Equation 12) and that for a self-shadowed disk

$$r_\tau \sim 1.2\mathrm{AU}\left(\frac{\alpha}{0.01}\right)^{\frac{2}{3}}\left(\frac{M_*}{M_\odot}\right)^{\frac{5}{3}} \tag{20}$$

(modifying Equation 8) Though the minimum gap opening planet mass is not significantly different in the self-shadowed disk than the flared one, lower mass planets can open gaps at larger radius.

If larger planets are formed earlier during epochs of higher accretion around more massive stars then we would predict that they might be more likely to be located at larger radii. This follows because the transition radius is larger for higher mass stars.

4 DISCUSSION AND SUMMARY

In this paper we have explored the relation between the minimum gap-opening planet mass and disk parameters, accretion rate, the alpha parameter and stellar mass. We have used simple models for α discs, with temperature set by heating due to viscous accretion and stellar irradiation.

We find that the minimum gap opening planet mass is not strongly sensitive to radius at radii where the disk is heated primarily by viscous dissipation and is optically thick However, the minimum gap opening planet mass increases with radius where the disk is heated primarily by stellar radiation. Due to the weak dependence with radius in an optically thick disk heated by viscous dissipation, the minimum mass gap opening planet in any disk is best estimated by the viscous criterion (Equation 13). For a self-shadowed disk, the minimum gap opening planet mass is similar to that for a flared disk, however the radius at which the mass begins to increase more rapidly is at larger radius and is approximately where the disc becomes optically thick.

We estimate that a 0.4 Jupiter mass planet is required to open a gap in a flared disk with $\dot{M} = 10^{-8} M_\odot/\mathrm{yr}$ and $\alpha = 0.01$ around a $1 M_\odot$ star. Lower mass planets can open gaps in disks with lower accretion rates and lower α parameters around a lower mass star. We estimate that the minimum gap opening planet mass is proportional to $\dot{M}^{0.48}\alpha^{0.8}M_*^{0.42}L_*^{-0.08}$ (Equation 17). This scaling relation can be used to place limits on planets residing in gapless disks as a function of accretion rate, α parameter, stellar mass and stellar luminosity for both flared and self-shadowed disks.

It is interesting to speculate on scenarios for planet migration. A planet that is not sufficiently massive to open a gap but continues to accrete and migrate could open a gap if it accretes mass sufficiently rapidly (Thommes et al. 2007) or because it reaches a location in the disk where migration ceases (Masset et al. 2006; Crida and Morbidelli 2007). Consider a planet which has formed far out in the disk, and is migrating inwards in the linear (Type I) regime. If it reaches the transition radius (r_{tr}) before achieving the mass required for gap opening at that point (Equation 16), then it will continue its rapid inward migration, and will probably be lost on to the star. The weak dependence of the gap opening mass in the inner disk with radius means that it is unlikely that the planet will be able to accrete fast enough to open a gap within r_{tr}. The transition radius represents a 'last chance' for a planet to open a gap and slow its migration. If a disk is observed without a gap, Equation 16 provides a bound on the most massive object in the inner portions of the disk.

The different models do predict different locations for planets to slow their migration. In the planet trap proposed by Masset et al. (2006) the truncation of the disk near the star sets the location of the planet trap. As accretion would not be detected after a planet formed, the scenario suggested here would relate the mass of the first planet formed to the minimum detectable disk accretion rate. If planet formation cuts off disk accretion and planets stop migration at the transition radius then we would predict a relation between clearing size and stellar mass. Equation 19 which relates the transition radius to the stellar mass assuming $L_* \propto M_*^2$ and $\dot{M} \propto M_*^3$ suggests that the radius of the clearing following the first gap opening planet would grow with the stellar mass. We note that the radii, r_{tr} and r_τ, estimated here are of order 1 AU at an accretion rate of $10^{-8}M_\odot$/yr and so smaller than hole clearing radii measured for objects such as CoKuTau/4. The transition radius is only as large as CoKuTau/4's ($\sim$ 10AU) at an accretion rate of order $\dot{M} \sim 10^{-7}M_\odot$/yr.

The disk models considered here are simplistic and do not include the radiative transfer of more sophisticated models such as D'Alessio et al. (2001); Garaud and Lin (2007) which could be used to improve upon the accuracy of our constraints on the minimum gap-opening planet mass. Better observational constraints on the α viscosity parameter would also improve these estimates. We have explored disks with opacity with different temperature laws for gas disks using the opacities described by Bell and Lin (1994); Bell et al. (1997) The shapes of the curves differ quantitatively but not qualitatively from those shown here. In each temperature regime the disk temperature has a different slope but the overall shape of the temperature, aspect ratio and density curves are similar to and within a factor of a few of those shown here. We have not shown these curves here as the dust opacity is expected to dominate that of the gas, and the simple model explored here can be solved more easily to produce our scaling relations.

Here we have only considered models of steady state disks with constant accretion rate $\dot{M}$. A starved disk that has not enough mass at large radius to maintain its accretion rate would have lower disk density at larger radius than predicted here. If the disk were flared, the disk temperature would be set by stellar irradiation and the minimum gap-opening planet mass estimate would not vary from what is predicted here. However were the disk to become self-

shadowed the minimum gap opening planet mass would be lower at than that predicted with a constant $\dot{M}$ (this follows because q_{min} decreases with decreasing $\dot{M}$; Equations 13 and 14).

Assuming that the accretion processes within the disc can be fully parameterised by α is questionable. The source of accretion disc viscosity is poorly understood, but the most likely candidate is the magneto-rotational instability (MRI). Winters et al. (2003) performed magneto-hydrodynamic (MHD) calculations of a planet embedded in a disc, and found that the gap structure was markedly different to that obtained from a calculation which included a physical viscosity. We have also neglected the possibility of a 'dead zone' in the disc (Gammie 1996), where the high midplane densities shut down the MRI. The surface layers continue to be ionised by cosmic rays, and can still accrete. Within the dead zone, the effective viscosity is expected to be dramatically lower, and hence much lower mass planets might be able to open gaps. In this case, the first (tidal) term of Equation 1 would become more significant in determining whether a planet can open a gap. This is in contrast to the results we have presented here, which are dominated by the second (viscous) term. Matsumura and Pudritz (2005, 2006) considered gap opening in a disc with a dead zone. Figure 2 of the 2005 paper shows their expected minimum mass gap opening planets, as a function of radius. Matsumura and Pudritz found that the minimum mass planet required to open a gap was generally increasing with radius (unlike the nearly constant value in the inner regions we calculated above), with a step at the boundary of the dead zone. There are numerous differences between their work and ours; not only are the disc models different, but also the gap opening criterion. A criterion calculated by Rafikov (2002) is used, which is based on a calculation of spiral shock dissipation in circumstellar discs. The structures formed by a planet which satisfies the viscous, but not the tidal, condition for gap opening (that is, the first term of Equation 1 is greater than 1, but the second term is much less than 1) have not been extensively studied numerically, and we intend to investigate them further in future work. It should be noted that 'dead zones' might not be dead as first thought. Although the MRI cannot operate (due to the low ionisation levels), turbulence can diffuse downwards from the surface layers (Fleming and Stone 2003; Oishi et al. 2007), stirring material and driving accretion.

In this paper, we have calculated the minimum mass planet capable of opening a gap in a steady state accretion disc. In the inner regions, the minimum mass required is very weakly dependent on radius. Further out, the minimum mass grows with radius. The transition between the two regimes (located where heating by stellar radiation dominates, or where a self-shadowed disc becomes optically thin) represents a 'last chance' for a rapidly migrating embedded planet to open a gap. If a planet is unable to do this, it is likely to be lost onto the star.

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